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Entropy of thermally excited black rings

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Abstract: A string theory description of near extremal black rings is proposed. The entropy is computed and the thermodynamic properties are derived for a large family of black rings that have not yet been constructed in supergravity. It is also argued that the most general black ring in $N = 8$ supergravity has 21 parameters up to duality.

Keywords: Black Holes, Black Holes in String Theory.

Contents

1. Introduction

The existence of black ring solutions [1] raises interesting questions in classical general relativity [2] and, since black rings are readily embedded in supergravity [3 − 5], in string theory. One important challenge is to understand black rings microscopically in string theory. For BPS black rings it was proposed that one can simply identify circular black rings with straight black strings [6 − 8]. This prescription gives a statistical account of the black ring entropy; but it also highlights some confusion about the precise distinction between black holes and black rings. Therefore, a more detailed microscopic description of black rings is essential even for the understanding of black holes.

The purpose of this paper is to propose a microscopic description of thermal black rings, i.e. rings that are excited away from the extremal limit. The strategy is, again, to identify circular black rings with straight black strings. It is well-known how to take thermal excitations of black strings into account, in the limit where the excitation energy is not too large [9] and we simply adapt this description to black rings. The only modest complication is due to the fact that black rings carry, in addition to the dipole charges along the circular string, several additional charges. These charged excitations arise as zero-modes of affine currents in the two dimensional effective CFT describing the collective excitations of the black string [10, 11]. Combining this with general principles, we find an expression for the entropy as function of energy, angular momenta, and all charges. Our final result for the entropy is given in (3.7) below. As a simple application of this result we work out the thermodynamic properties of black rings.

Thermally excited black rings are much richer than their extremal counterparts. This will further focus attention on the limitations of describing black rings in terms of black strings. As is clear already in the extremal limit, the identification of these two theories is valid only near the objects, where the extrinsic curvature of the black ring can be neglected. Such a description is incomplete because, from the dual open string perspective, it applies only in the infra-red, i.e. it is an effective description. In particular, this means the parameters of the microscopic description cannot immediately be identified with the corresponding quantities in the supergravity solution (some relevant discussions are [12, 13]). This is familiar already for BPS rings where several different definitions of charges and angular momenta seem relevant for the description. In the excited theory these ambiguities persist and, in addition, the microscopic energy must be distinguished from the mass of the black ring. Of course, such features do not invalidate the proposed microscopic descriptions; they are properties of effective theories.

A by-product of the present investigation is the determination of duality orbits for black rings. For black holes in five dimensions it is sufficient to consider supergravity solutions with 3 charges because, starting with such a solution, dualities can generate the most general black hole, characterized by 27 conserved charges [14]. The most general black ring in five dimensions depends on 27 charges as well, but also on 27 dipole charges. We will argue that, to generate such general rings, the starting point must have 3 charges and 15 dipole charges (or *vice versa*). Taking mass and angular momenta into account, the most general black ring would then have 21 parameters. This is a much larger class than those already constructed in the BPS case [5, 4], and also much larger than those previously conjectured in the non-BPS case [15].

The microscopic interpretation discussed in this paper gives predictions for supergravity solutions that have not yet been constructed explicitly:

1. The area of a conjectured 9 parameter family of thermally excited black rings is identified.

2. There exists an 8 parameter family of *extremal* black rings. These black rings are not supersymmetric, but they are extremal in the sense that they have vanishing temperature.

3. Thermal black rings are expected to have an inner and an outer horizon, both of topology $S^1 \times S^2$. The area of the *inner* horizon is predicted as well.

The remainder of this paper is organized as follows. In section 2 we discuss the action of string dualities on black rings and the parameters needed to describe the most general black ring solution in five dimensions. section 3 is the core of the paper: we develop the microscopic description of the black rings, by adapting the description of black strings. In section 4 we discuss the resulting thermodynamics of black rings. We conclude in section 5 with a brief discussion of the ambiguities in the definition of charges.

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2. Black rings and dualities

At the classical level the theory we discuss is 11 dimensional supergravity compactified on T^6 or, equivalently, $N = 8$ supergravity in five dimensions. The issue we want to address is that, in this theory, duality relates apparently distinct configurations, effectively reducing the number of truly independent solutions[1]. In order to factor out this redundancy in the description we want to determine the duality orbits of black ring solutions.

It is convenient to parametrize solutions in supergravity by their asymptotics near infinity. Concretely, this means specifying gravitational mass M and angular momenta J_a ($a = 1, 2$), the asymptotic value of all scalar fields X_I^∞ ($I = 1, \ldots, 42$), and the charges of gauge fields. The gauge charges Q_I may be computed by gaussian flux integrals[2]

$$Q_I = \frac{1}{2\pi^2} \int_\infty G_{IJ} {}^*F^I \tag{2.1}$$

In the case of black rings there are also the dipole charges q^I specified by integrals

$$q^I = \frac{1}{2\pi} \int_{S^2} F^I \tag{2.2}$$

where the integral is over a sphere that links the ring. The dipole charges fall off faster asymptotically than the ordinary charges, and they are not conserved; but this does not make them any less useful in the classification.

The duality group of $N = 8$ supergravity in $D = 5$ is $E_{6(6)}(R)$. The 42 scalar fields X^I parametrize the coset $E_{6(6)}/\,\mathrm{USp}(8)$ (dimension $78 - 36 = 42$). The values of these scalars at infinity can be chosen arbitrarily; they are just integration constants. Indeed, as indicated by the coset form of the scalar manifold, different asymptotic values of the scalars are related by $E_{6(6)}(R)$ transformations. Making a definite choice, e.g. taking X^I_∞ corresponding to a square torus with no fluxes and sides of unit length, defines the vacuum and breaks the symmetry spontaneously as $E_{6(6)} \to \mathrm{USp}(8)$.

The 27 gauge fields in the theory transform in the antisymmetric symplectic traceless representation of the USp(8) symmetry leaving the vacuum invariant. To make this explicit, it is convenient to organize the 27 gauge field charges into the 8×8 central charge matrix [18]

$$Z_e = \begin{pmatrix} BJ_{(1)} & A \\ -A^T & -\frac{1}{3} BJ_{(3)} + C_{ij} T^{ij} \end{pmatrix} \tag{2.3}$$

where $J_{(i)}$ are symplectic invariants of $\mathrm{USp}(2i)$ and T^{ij} are a basis of trace-less anti-symmetric 6×6 matrices. One can choose the duality frame so that the 2×6 charges A correspond to $M5$-branes and KK-waves wrapped on cycles fully within the T^6. Then the

[1] We are just discussing dipole charges, in addition to the conventional black hole charges. It is possible that black holes and black rings in five dimensions support additional classical hair; indeed, such hair could ultimately account for the entire microscopic structure of black holes, as recently advocated by Mathur and collaborators [16] (and earlier in [17]).

[2] We use units where the 11-dimensional planck length $l_p = (\pi/4G_5)^{1/3} = 1$. In these units all charges are quantized.

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B, C_{ij} are 15 charges corresponding to $M2$-branes wrapped on the two-cycles of the T^6. Acting by the USp(8) duality group, the central charge matrix can be skew-diagonalized. In the canonical duality frame just introduced, this amounts to turning on just three charges, interpreted as M2-branes wrapped on the (12), (34), and (56) cycles of the T^6.

The important point is that a reference solution with just these three charges is in fact the most general one, up to duality. Let us show explicitly how duality can reintroduce all charges: the reference solution, with skew-diagonal charge matrix, is left invariant by a subgroup $\mathrm{US}(2)^4 \subset \mathrm{USp}(8)$. New solutions are thus found by acting with $\mathrm{USp}(8)/\,\mathrm{US}(2)^4$ on the reference solution . This amounts to adding $36 - 4 \times 3 = 24$ parameters, recovering the general charge configuration with $3 + 24 = 27$ parameters [14].

Next, we discuss the dipole charges, the novel feature introduced by black rings. These are "magnetic" charges of the same 27 gauge fields considered above, but they are string-like in character, rather than the point-like "electric" charges parametrized in (2.3). The dipole charges can similarly be organized into a magnetic "central charge" matrix

$$Z_m = \begin{pmatrix} B_m J_{(1)} & A_m \\ -A_m^T & -\frac{1}{3} B_m J_{(3)} + C_{m\ ij} T^{ij} \end{pmatrix} \tag{2.4}$$

which, again, is anti-symmetric and symplectic traceless under the USp(8) duality in a given vacuum. In the canonical duality frame introduced after (2.3), the 2×6 charges A_m are the 6 $M2$-branes and the 6 KK-monopoles with one direction transverse to the T^6; and the B_m, $C_{m\ ij}$ jointly describe the $M5$-branes wrapping the 15 independent four-cycles within the T^6. Again, the magnetic central charge matrix Z_m can be skew-diagonalized by a suitable USp(8) duality transformation. However, in the duality frame where the electric charge matrix Z_e has already been simplified in this manner, the magnetic charges are *not* in general diagonal.

The subgroup $\mathrm{US}(2)^4 \subset \mathrm{USp}(8)$ that leaves the skew-diagonal Z_e invariant in general acts non-trivially on Z_m, generating 12-parameter orbits of equivalent solutions. Concretely, we can choose $A_m = 0$ without loss of generality, i.e. keep only the 15 dipole $M5$-brane charges. The 6 dipole $M2$-branes and the 6 dipole KK-monopoles can be taken to vanish because these are generated when $\mathrm{US}(2)^4$ duality transformations act on the $M5$-brane dipoles.

In summary, we have shown that the most general black ring in $N = 8$ supergravity is parametrized up to duality by 21 parameters: the mass M, 2 angular momenta J_a ($a = 1, 2$), 3 eigenvalues Q_i ($i = 1, 2, 3$), and 15 dipole charges B_m $C_{m\ ij}$. As a check note that the total number of black ring parameters is $3 + 27 + 27 = 57$ (from gravitational, point-like, and string-like charges). Since there are 36 USp(8) duality parameters in a given vacuum we find that a seed solution must have $57 - 36 = 21$ parameters, as in the more detailed argument.

In the following we will for simplicity focus on the "canonical" 9 parameter family of black rings where the electric and magnetic central charge matrices (2.3)-(2.4) are simultaneously diagonalized. This configuration is left invariant by $\mathrm{US}(2)^4 \subset \mathrm{USp}(8)$ so, when acting on these solutions, duality transformations can only add the 24 parameters of the coset $\mathrm{USp}(8)/\mathrm{US}(2)^4$. The canonical 9 parameter family of rings therefore correspond, after duality is taken into account, to completely general charges, but only three dipole charges.

3. The microscopic theory

In this section we discuss some features of the microscopic description of the black rings.

3.1 General comments

For extremal black rings it has been proposed that the effective low energy theory of the collective excitations is identical to the two dimensional CFT with $(4,0)$ supersymmetry governing black holes in *four* dimensions [6 − 8]. Some motivations for this identification are:

1. *Horizon topology*: the $S^2 \times S^1$ topology of the black ring horizon is identical to that of a five dimensional black string which, in a suitable limit, is interpreted as a four dimensional black hole. Then the S^2 of the ring is identified with the horizon of the black hole and the S^1 of the ring corresponds to the compact dimension.

2. *Near horizon geometry:* in the limit where bulk gravity decouples from the theory on the branes, the near horizon geometry of the standard black holes is locally $AdS_3 \times S^2$, and the global structure is that of an extremal BTZ black hole. AdS/CFT correspondence then identifies the miscroscopic theory [20]. Black rings similarly allow a near string limit which decouples bulk modes and identifies a near horizon BTZ black hole [22, 5, 7]. This shows that the two microscopic theories are identical as well.

3. *The effective string*: the usual description of 4D black holes involves an asymmetric scaling limit that singles out one compact dimension, which is interpreted as the spatial direction of an effective string [19, 11]. It is natural to simply identify this effective string with the black ring. Again, this is because the geometry near the ring is indistinguishable from that of a straight string.

The working hypothesis of this paper is that 5D black rings can be identified with 5D black strings (and so with 4D black holes) also in the non-extremal case, as long as the excitations above the extremal limit remains small. This assumption is natural because the arguments above for the extremal case remain valid (to the extent we can check them).

3.2 The CFT description

In the canonical duality frame described after (2.3) the black string consists of is $M5$-branes with wrapping numbers q^1, q^2, q^3 along the cycles orthogonal to the canonical (12), (34), (56) two-cycles. These five-branes all share one common line which is the locus of the effective two dimensional CFT. This spacetime CFT has central charge $c = 6q^1q^2q^3$ for both right and left movers and $(4,0)$ supersymmetry. The $M2$ branes (and all other charges) are realized as charged excitations of this theory.

The supersymmetric sector of the CFT are the right movers. The $N = 4$ superconformal algebra contain an affine US(2) current at level $k = \hat{c} = \frac{2}{3}c = 4q^1q^2q^3$. States with quantum number J under a U(1) subgroup of this R-symmetry have level

$$h_R^{\rm rot} = \frac{1}{k}j^2 = \frac{1}{4q^1q^2q^3}J^2\,. \tag{3.1}$$

The US(2) symmetry is interpreted in spacetime as the rotation group in the three dimensional transverse space; so the quantum number J is simply the projection of the angular momentum along the quantization axis.

There is one additional right moving current. This current carries the charges dual to the charges defining the background. Concretely, spacetime supersymmetry gives the BPS mass[3]

$$M^2 = (q^1X_1 + q^2X_2 + q^3X_3)^2R^2 + (Q_1X^1 + Q_2X^2 + Q_3X^3)^2 \tag{3.2}$$

where X_I parametrize the volumes of the four-cycles wrapped by the $M5$-branes, R is the radius of the direction along the effective string, and the $X^I = 1/X_I$ correspond to the volumes of the dual two-cycles wrapped by the $M2$-branes. Treating the $M5$-branes as a heavy background, the energy associated with the $M2$-brane excitations is

$$\Delta M \simeq \frac{1}{2(q^1X_1 + q^2X_2 + q^3X_3)R}(Q_1X^1 + Q_2X^2 + Q_3X^3)^2\,. \tag{3.3}$$

In the decoupling limit where the CFT applies this equality becomes exact. If excitations with this energy arise as zero-modes of affine currents the conformal weight associated with the charge is

$$h_R^{\rm M2} = \frac{R}{2}\Delta M = \frac{1}{12q^1q^2q^3}(Q_1q^1 + Q_2q^2 + Q_3q^3)^2 \tag{3.4}$$

where we let the scalar fields attain their attractor values $X^I = q^I/(q^1q^2q^3)^{1/3}$ for $I = 1, 2, 3$ (for a discussion of 5D attractors emphasizing black rings see [21]).

The remaining two linear combinations of $M2$-brane charges are not affected by spacetime supersymmetry; so these are carried by left moving currents. The levels of these currents are constrained by modular invariance of the CFT which relates right and left moving currents. The simplest possibility is to form a lattice of signature $(2, 2)$ with the right moving currents [11]. This prescription gives the conformal weights of the zero-modes

$$\begin{aligned} h_L^{\rm M2} &= \frac{1}{4q^1q^2q^3}(Q_1q^1 - Q_2q^2)^2 + \frac{1}{12q^1q^2q^3}(Q_1q^1 + Q_2q^2 - 2Q_3q^3)^2 \\ &= \frac{1}{3q^1q^2q^3}\left[(Q_1q^1)^2 + (Q_2q^2)^2 + (Q_3q^3)^2\right] - \\ &\quad -\frac{1}{3q^1q^2q^3}\left[Q_1q^1\cdot Q_2q^2 + Q_1q^1\cdot Q_3q^3 + Q_2q^2\cdot Q_3q^3\right]\,. \end{aligned} \tag{3.5}$$

Since (3.5) is not protected by supersymmetry this expression can only be trusted in the semi-classical regime.

[3]In our units the brane tensions are automatically taken into account correctly.

It is not difficult to generalize the argument leading to (3.4) and (3.5) to find a lattice of signature $(14, 14)$ that describes all 27 charged excitations and angular momentum. The result requires a bit more notation because it depends on the complex structure moduli so we just refer to [11] for the details. The microscopic theory describes the most general thermal black ring up to duality when this full set of excitations is taken into account.

The arguments presented for the existence of the various currents and for the weight of their zero-modes is somewhat heuristic; but there are several checks on the assignments:

1. *Triality*: the formulae (3.4) and (3.5) for the weights are invariant under simultaneous permutations of the $M5$ brane charges and the $M2$ brane charges. This symmetry is in the Weyl subgroup of the duality group and must be respected.

2. *Rational levels*: the levels (3.4) and (3.5) turned out to be rational even though the attractor mechanism fixes the scalars at irrational values $X^I = q^I/(q^1q^2q^3)^{1/3}$.

3. *Level matching*: the difference between levels

$$\begin{aligned} h_L^{\mathrm{M2}} - h_R^{\mathrm{M2}} &= \frac{1}{4q^1q^2q^3}\left[(Q_1q^1)^2 + (Q_2q^2)^2 + (Q_3q^3)^2\right] - \\ &\quad -\frac{1}{2q^1q^2q^3}\left[Q_1q^1 \cdot Q_2q^2 + Q_1q^1 \cdot Q_3q^3 + Q_2q^2 \cdot Q_3q^3\right] \end{aligned} \tag{3.6}$$

   is quantized in the same unit $1/k = 1/4q^1q^2q^3$ as the angular momentum. This means the higher modes of currents can be matched consistently.

4. *Global symmetry*: the total set of 27 currents in the theory (discussed after (3.5) and in section 2) transform as $(\mathbf{1}, \mathbf{1}) \otimes (\mathbf{2}, \mathbf{6}) \otimes (\mathbf{1}, \mathbf{14})$ under the duality $\mathrm{US}(2) \times \mathrm{USp}(6) \subset \mathrm{USp}(8)$ preserved by the fixed scalar conditions. The $(\mathbf{1}, \mathbf{1}) \otimes (\mathbf{2}, \mathbf{6})$ are right movers, with level determined by the spacetime BPS algebra, as discussed before (3.4). The $(\mathbf{1}, \mathbf{14})$ relates the normalizations of *all* left moving currents [11] and confirm (3.5).

3.3 Results

We now count the entropy from the degeneracy of the states in the CFT. The vertex operators of the states we are counting are given by

$$\mathcal{V}_{\mathrm{tot}} = \mathcal{V}_{\mathrm{irr}}\mathcal{V}_{\mathrm{U}(1)}\,.$$

The $\mathcal{V}_{\mathrm{U}(1)}$ is constructed from the U(1) currents such that the full vertex operator carries the correct U(1) charge. The conformal weight accounted for by this was derived above. The $\mathcal{V}_{\mathrm{irr}}$ can be specified freely and so gives rise to entropy. The expression for the entropy is given as usual by Cardy's formula

$$S = 2\pi\left[\sqrt{\frac{ch_L^{\mathrm{irr}}}{6}} + \sqrt{\frac{ch_R^{\mathrm{irr}}}{6}}\right] \tag{3.7}$$

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where presently the irreducible weights are given

$$h_L^{\text{irr}} = \frac{\epsilon + p}{2} - \frac{1}{3q^1q^2q^3}\left[(Q_1q^1)^2 + (Q_2q^2)^2 + (Q_3q^3)^2\right] + \\ + \frac{1}{3q^1q^2q^3}\left[Q_1q^1 \cdot Q_2q^2 + Q_1q^1 \cdot Q_3q^3 + Q_2q^2 \cdot Q_3q^3\right] \tag{3.8}$$

$$h_R^{\text{irr}} = \frac{\epsilon - p}{2} - \frac{1}{12q^1q^2q^3}\left[Q_1q^1 + Q_2q^2 + Q_3q^3\right]^2 - \frac{1}{4q^1q^2q^3}J^2 \,. \tag{3.9}$$

This is our final result for the entropy. In this formula the momentum quantum number p along the black string should be identified with the angular momentum along the black ring and the four-dimensional angular momentum J is the angular momentum transverse to the ring $J = J_\phi$.

The extremal limit is given by $h_R^{\text{irr}} = 0$. In this limit we can eliminate ϵ and find

$$h_L^{\text{irr}} = p - \frac{1}{4q^1q^2q^3}\left[(Q_1q^1)^2 + (Q_2q^2)^2 + (Q_3q^3)^2\right] + \\ + \frac{1}{2q^1q^2q^3}\left[Q_1q^1 \cdot Q_2q^2 + Q_1q^1 \cdot Q_3q^3 + Q_2q^2 \cdot Q_3q^3\right] + \frac{1}{4q^1q^2q^3}J^2 \,. \tag{3.10}$$

If in addition we impose supersymmetry then $J = 0$. In this BPS limit our general result (3.7) for the entropy reduces to the one deduced from $E_{7(7)}$ duality symmetry of the black string in [7, 25], and from counting deformations of $M5$-branes in [8, 26]. The 8 parameter family of configurations with $h_R^{\text{irr}} = 0$ but $J \neq 0$ corresponds to black rings that are extremal, but not supersymmetric.

The general 9-parameter family of near extremal black rings have not yet been constructed in supergravity. The formula (3.7) for their entropy predicts the area of their outer horizon

$$A_+ = 2\pi^2 \, \frac{1}{2}\left(\sqrt{h_L^{\text{irr}}} + \sqrt{h_R^{\text{irr}}}\right) . \tag{3.11}$$

These black rings are expected to have an inner horizon as well, also of topology $S^1 \times S^2$ [23, 15]. The considerations of [24] predicts the area of the *inner* horizon

$$A_- = 2\pi^2 \, \frac{1}{2}\left(\sqrt{h_L^{\text{irr}}} - \sqrt{h_R^{\text{irr}}}\right) \tag{3.12}$$

where the $h_{L,R}^{\text{irr}}$ are given in (3.8) and (3.9).

4. Black ring thermodynamics

In this section we derive the thermodynamics of the near extremal black ring. The strategy is to treat the entropy (3.7) as a potential that generates all other physical features of the ring through the first law of thermodynamics

$$dM = TdS + \Omega_a dJ^a + \Phi^I dQ_I + \varphi_I dq^I \,. \tag{4.1}$$

Here M is the total mass of the ring, T is the temperature of Hawking radiation and S is the entropy. The J^a with $a = 1, 2$ are the two angular momenta and the Ω_a are the corresponding potentials, interpreted geometrically as the rotational velocities at the horizon. The M2-brane charges are Q_I, and the Φ^I are the corresponding electromagnetic potentials at the horizon. Finally the M5-brane dipole charges are q^I, and φ_I are the corresponding magnetic potentials at the horizon.

The first law readily gives the temperature through

$$\frac{1}{T} = \left(\frac{\partial S}{\partial M}\right)_{J_a, Q_I, q^I} . \tag{4.2}$$

We find

$$\frac{1}{T} = \frac{1}{T_R} + \frac{1}{T_L} \tag{4.3}$$

where

$$T_{L,R} = \frac{2}{\pi}\sqrt{\frac{6h^{\rm irr}_{L,R}}{c}} . \tag{4.4}$$

The two temperatures T_L and T_R are interpreted as usual as the independent temperatures of the left and right moving excitations.

In the extremal limit $h^{\rm irr}_R \to 0$ we have $T_R \to 0$ so that right moving excitations are forced to the ground state. This takes the spacetime temperature $T \to 0$ as well. On the other hand, the left moving temperature approaches the finite value

$$T_L \to \frac{12}{\pi c}\sqrt{\frac{ch^{\rm irr}_L}{6}} = \frac{6}{\pi^2 c} S \tag{4.5}$$

in the extremal limit. This is the temperature of the highly degenerate ground state responsible for the entropy.

A natural parametrization for the two angular momenta $a = 1, 2$ is to identify one index with the momentum along the ring $a = p$ and the other with the angular momentum $J_\phi = J$ in the plane transverse to the ring. We then find

$$\frac{\Omega_p}{T} = -\left(\frac{\partial S}{\partial p}\right)_{M, J_\phi, Q_I, q^I} = \frac{\pi}{2}\sqrt{\frac{c}{6}}\left[\frac{1}{\sqrt{h^{\rm irr}_R}} - \frac{1}{\sqrt{h^{\rm irr}_L}}\right] = \frac{1}{T_R} - \frac{1}{T_L} \tag{4.6}$$

for the rotational velocity along the ring; and

$$\frac{\Omega_\phi}{T} = -\left(\frac{\partial S}{\partial J}\right)_{M, p, Q_I, q^I} = \frac{\pi}{2}\sqrt{\frac{c}{6h^{\rm irr}_L}}\frac{J}{q^1 q^2 q^3} = \frac{\pi}{2}\sqrt{\frac{6}{ch^{\rm irr}_L}} J = \pi^2 \frac{J}{S_L} \tag{4.7}$$

for the rotational velocity transverse to the ring. Here S_L is the entropy of the left-movers.

The rotational velocity can never exceed the speed of light $\Omega_p < 1$ as this would, effectively, amount to the development of closed time-like curves. However, in the extremal limit, the rotational velocity along the direction of the ring approaches the speed of light as

$$\Omega_p = \frac{T_L - T_R}{T_L + T_R} \to 1^- . \tag{4.8}$$

On the other hand, the rotational velocity in the plane transverse to the ring slows down $\Omega_\phi \to 0$. This happens at the same rate as $T \to 0$ such that, in the extremal limit, there can be a finite ratio

$$\frac{\Omega_\phi}{T} \to \pi^2 \, \frac{J}{S} \, . \tag{4.9}$$

Of course BPS black holes have $J = 0$.

For completeness, let us also consider the electromagnetic potentials. They are

$$\begin{aligned}
\frac{\Phi^1}{T} &= -\left(\frac{\partial S}{\partial Q_1}\right)_{M,J_a,Q_{2,3},q^I} \\
&= \frac{\pi}{3q^1q^2q^3}\sqrt{\frac{c}{6}}\left(\frac{2Q_1q^1 - Q_2q^2 - Q_3q^3}{\sqrt{h_L^{\mathrm{irr}}}} + \frac{Q_1q^1 + Q_2q^2 + Q_3q^3}{2\sqrt{h_R^{\mathrm{irr}}}}\right) q^1 \\
&= \frac{4}{c}\left(\frac{2Q_1q^1 - Q_2q^2 - Q_3q^3}{T_L} + \frac{Q_1q^1 + Q_2q^2 + Q_3q^3}{2T_R}\right) q^1
\end{aligned} \tag{4.10}$$

for the usual charges and

$$\begin{aligned}
\frac{\varphi_1}{T} &= -\left(\frac{\partial S}{\partial q^1}\right)_{M,J_a,Q_I,q^{2,3}} \\
&= \frac{\pi}{3q^1q^2q^3}\sqrt{\frac{c}{6}}\left(\frac{2Q_1q^1 - Q_2q^2 - Q_3q^3}{\sqrt{h_L^{\mathrm{irr}}}} + \frac{Q_1q^1 + Q_2q^2 + Q_3q^3}{2\sqrt{h_R^{\mathrm{irr}}}}\right) Q^1 + \\
&\quad + \pi\left(\sqrt{\frac{c}{6h_L^{\mathrm{irr}}}}\frac{\epsilon + p}{2q^1} + \sqrt{\frac{c}{6h_R^{\mathrm{irr}}}}\frac{\epsilon - p}{2q^1}\right) \\
&= \frac{4}{c}\left(\frac{2Q_1q^1 - Q_2q^2 - Q_3q^3}{T_L} + \frac{Q_1q^1 + Q_2q^2 + Q_3q^3}{2T_R}\right) Q_1 + \frac{\epsilon - \Omega_p p}{q^1 T}
\end{aligned} \tag{4.11}$$

for the dipole charges. The remaining potentials $\Phi^{2,3}$ and $\varphi_{2,3}$ are found by the obvious cyclic permutations.

In the extremal limit $T_R \to 0$ and $\epsilon \to p$ with T_L finite so that

$$\Phi^I \to \frac{4}{c}\left(Q_J q^J\right) q^I \tag{4.12}$$

$$\varphi_I \to \frac{4}{c}\left(Q_J q^J\right) Q_I \, . \tag{4.13}$$

Note that, in the extremal limit, the electric and magnetic potentials are related such that

$$\Phi^I Q_I = \varphi_I q^I \qquad \text{(no sum over I)} \, . \tag{4.14}$$

5. Discussion

As we have emphasized repeatedly, the strategy in this paper is to find a microscopic theory of black rings by identifying the circular black rings with straight black strings. The results are formulae for the thermodynamics of objects that seem quite difficult to construct even at the level of supergravity. As is clear from the derivation, the results are written in the variables that are natural for the black string. When the results are interpreted in terms of the black ring, as advocated here, these variables must be defined in the near horizon geometry; and so they may in general differ from the corresponding variables in the asymptotic geometry.

The distinction between near horizon and asymptotic variables is familiar already from the extremal case. The near horizon charges Q_I used in the present paper can be identified with the "barred" charges $\bar{Q}_I$ of [7, 12] and should be distinguished from asympotic charges of the asymptotically flat black ring [5, 8]. Similarly, as in [7, 12], the momentum along the ring is related to the angular momenta in the asymptotic space as $p = -(J_\psi + J_\phi)$ (in the notation of [5]).

For the thermal ring there is an additional ambiguity: the excitation energy of the microscopic theory plays the role of "mass" in the near horizon geometry but this mass, and its dual temperature, cannot be identified with the mass parameter in the asymptotically flat space. The distinction is seen clearly by noting that the black ring description associates a non-vanishing energy with *all* charged excitations, e.g. in (3.2). In contrast, the BPS mass in the asymptotically flat space is independent of dipole charges, through the mechanism familiar from supertubes [27].

The microscopic definitions of the parameters used here are clear close to the horizon: the near horizon geometry of the ring (not yet constructed explicitly) is expected to be AdS_3, because this is the geometry of the corresponding string. The charges, angular momenta, and mass are the parameters defined asymptotically in this space; and this is also where the central charge of the dual theory is defined, as are the levels of the various affine currents exploited in this paper.

It would clearly be interesting to complement the description pursued here with an interpretation of black rings as excitations in the asymptotically flat geometry. This would also help clarify the relation between black rings and black holes in five dimensions.

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